Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0123456789

FURTHER MATHEMATICS **9231/03**

Paper 3 Further Mechanics **For examination from 2020**

SPECIMEN PAPER **1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **14** pages. Blank pages are indicated.

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1 A child's toy consists of a uniform solid circular cone, of vertical height $3r$ and radius r, and a uniform solid hemisphere of radius r. The circular bases of the cone and the hemisphere are joined together so that they coincide. The cone and the hemisphere are made of the same material.

Show that the centre of mass of the toy is at a distance $\frac{27}{10}r$ from the vertex of the cone. [4]

2 A light elastic string has natural length a and modulus of elasticity $24mg$. One end of the string is attached to a fixed point A. The other end of the string is attached to a particle of mass $2m$.

(a) Find, in terms of a, the extension of the string when the particle hangs freely in equilibrium below A. [2]

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(b) The particle is released from rest at A.

Find, in terms of a, the distance of the particle below A when it first comes to instantaneous rest. [6]

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3 A particle P of mass m kg falls from rest under gravity. There is a resistive force of magnitude mkv^2 N, where v m s^{-1} is the speed of P after it has fallen a distance x m and k is a positive constant.

(a) By solving an appropriate differential equation, show that

$$v^2 = \frac{g}{k}(1 - e^{-2kx}).$$ [7]

It is now given that $k = 0.01$. The speed of P when x becomes large approaches $V \text{ m s}^{-1}$.

(b) **(i)** Find V correct to 2 decimal places. [1]

(ii) Hence find how far P has fallen when its speed is $\frac{1}{2}V \text{ m s}^{-1}$. [2]

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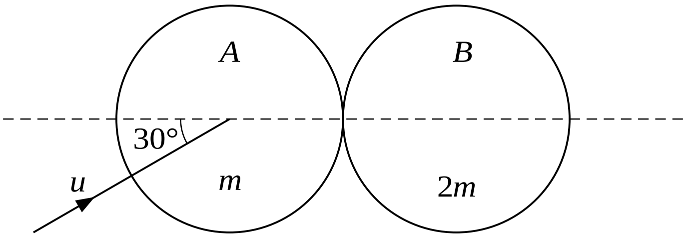

Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u at an angle of 30° to the line of centres of A and B when it collides with B (see diagram). The coefficient of restitution between the spheres is e.

(a) Show that the speed of B after the collision is $\frac{\sqrt{3}}{6}u(1+e)$ and find the speed of A after the collision. [6]

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(b) Given that $e = \frac{1}{3}$, find the loss of kinetic energy as a result of the collision. [3]

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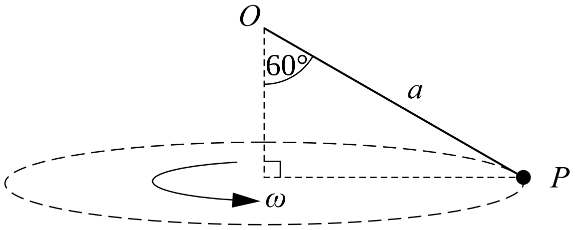
5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O.

(a)



The particle P moves in a horizontal circle with a constant angular speed ω with the string inclined at 60° to the downward vertical through O (see diagram).

Show that $\omega^2 = \dfrac{2g}{a}$. [4]

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(b) The particle now hangs at rest a distance a vertically below O. It is then projected horizontally so that it begins to move in a vertical circle with centre O. When the string makes an angle of 60° with the downward vertical through O, the angular speed of P is $\sqrt{\frac{2g}{a}}$. The string first goes slack when OP makes an angle θ with the upward vertical through O.

Find the value of $\cos\theta$. [6]

6 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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(b) The greatest height of P above the plane is denoted by H. When P is at a height of $\frac{3}{4}H$, it has travelled a horizontal distance d.

Given that $\tan\alpha = 2$, find, in terms of H, the two possible values of d. [6]

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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.